

The Visualisation–Construction–Reasoning (VCR) Instructional Model for School Geometry: A Design Grammar for Moving from Seeing to Justifying

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Abstract

Geometry instruction often struggles to connect three essential forms of mathematical work: disciplined noticing of structure, constraint-based production of geometric objects, and warrant-based justification of claims. As a result, students may rely on diagram appearance rather than invariant properties, treat construction as routine drawing, and experience reasoning as a disconnected formal demand. This paper introduces the Visualisation–Construction–Reasoning (VCR) Instructional Model as a geometry-specific instructional design grammar intended to address that coordination problem. The paper positions VCR in relation to adjacent traditions in mathematics education, including representational perspectives, visualisation research, construction-oriented approaches, and scholarship on proof and justification. It then defines the model conceptually and operationally, articulates its programme theory, and specifies its design principles, task ecology, sequencing rules, predictable error patterns, and teacher role shifts. The paper further describes the implementation architecture of the model through an 8-week, 35-lesson, bilingual, multimodal lesson-plan system and illustrates its classroom functioning through three flagship exemplars: altitude in a triangle, angle relations in parallel lines cut by a transversal, and area derivation through decomposition. The article also frames VCR as a replicable instructional package supported by structured lesson-plan validation evidence from expert and school-teacher panels. The contribution of the paper lies in presenting VCR not as a generic teaching strategy, but as a coherent, operationally specified instructional framework that links visualisation, construction, and reasoning into a pathway from perceptual engagement to mathematical warrant. The paper concludes by discussing the conceptual, methodological, and practical significance of the model, together with its limitations and the research agenda it enables.

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1. Introduction

Geometry occupies a distinctive place in school mathematics because it brings together spatial reasoning, representation, and mathematical argument in unusually visible ways (Clements & Battista, 1992; Jones, 2012; National Council of Teachers of Mathematics [NCTM], 2000). Learners are expected to inspect figures, identify patterns, construct objects under stated conditions, and justify claims about relationships among shapes, angles, lines, and measurements. In principle, geometry should therefore function as an ideal domain for linking intuition with disciplined reasoning. In practice, however, that coherence is often weak. Many students rely on the appearance of a diagram as sufficient evidence, interpret construction as a procedural drawing exercise, and experience proof as an externally imposed formality rather than as a meaningful extension of mathematical sense-making.

This problem is not simply the familiar claim that geometry is difficult. Rather, it reflects a recurring failure of classroom instruction to coordinate three forms of mathematical work: disciplined seeing, constraint-based building, and justification-based reasoning. Visual activity, although indispensable, does not automatically become mathematical

understanding, because learners must coordinate what is seen with mathematically relevant relations and representations (Duval, 2006; Laborde, 2005). A figure may appear symmetrical, equal, parallel, or perpendicular even when those properties have not been established mathematically (Duval, 2006; Fischbein, 1993). At the same time, proof is often introduced in ways that are disconnected from representational and constructive activity, so that students are asked to justify claims without having first experienced how those claims emerged from structured noticing and constrained action (Harel & Sowder, 2007; Stylianides, 2007). The result is a familiar split: on one side, visually rich but weakly disciplined activity; on the other, formal reasoning that lacks experiential grounding.

Research on visual representation has repeatedly shown that visualisation in mathematics is both process and product, and that its educational value depends on how learners are guided to create, interpret, coordinate, and reflect upon visual objects rather than merely look at them (Arcavi, 2003; Duval, 1995, 1998, 2006). Similarly, research on proof and proving has shown that justification should not be treated as an isolated terminal topic, but as a meaningful mathematical practice requiring clear norms for what counts as an acceptable argument in school mathematics (de Villiers, 1990; Stylianides, 2007). The challenge, then, is not whether visualisation or proof matters. It is how instruction can organise movement from one to the other in a principled and teachable way.

The present paper addresses that coordination problem by introducing the Visualisation–Construction–Reasoning (VCR) Instructional Model as a geometry-specific instructional design grammar. The model is based on the proposition that effective geometry teaching must deliberately organise learning through three linked forms of mathematical work. The first is visualisation, understood not as passive looking but as disciplined noticing of structure, invariance, variation, and spatial relations across diagrams and representations. The second is construction, understood not as decorative drawing but as the production and transformation of geometric objects under explicit constraints, with validation of those constraints. The third is reasoning, understood as the justification of claims through accepted properties, evidence, and connected argument rather than mere answer production.

VCR is not proposed as a generic teaching method, a fixed psychological sequence, or a proof-only pedagogy. It is proposed as a design grammar for instruction: a model that specifies what kinds of tasks should be used, how they should be sequenced, what predictable difficulties they should anticipate, what artefacts they should generate, and how teacher discourse should shift across phases. The model is iterative in enactment but structured in design. Reasoning may trigger renewed visualisation; failed construction may expose an unstable conjecture; a new representation may reorganise earlier understanding. What remains stable is the instructional movement from appearance-based judgement toward property-based construction and then toward warranted justification.

This article is framed as a conceptual–method paper. Its purpose is not to report treatment effects, test-score comparisons, or inferential outcomes. Rather, it aims to provide the flagship conceptual and operational definition of the VCR model. Specifically, the paper defines the model, positions it in the field, articulates its programme theory, specifies its design principles, outlines its task ecology, proposes sequencing rules, identifies predictable error patterns with diagnostic responses, presents its implementation architecture, and illustrates its classroom functioning through flagship exemplars. It also incorporates the lesson-plan validation and packaging logic that make the model usable as a standardised instructional system.

The article makes four main contributions. First, it offers a theoretically anchored but instructionally usable account of how visualisation, construction, and reasoning can be systematically coordinated in school geometry. Second, it advances VCR as a replicable model by specifying operative design rules, task families, sequencing constraints, and teacher role shifts. Third, it reframes geometry teaching as representational and constraint-based work rather than as the transmission of isolated propositions and procedures. Fourth, it provides an implementation-ready architecture that links conceptual design to lesson planning, multimodal resources, and validation-oriented packaging. These contributions are intended to make the paper useful both to researchers seeking a stable model for cumulative inquiry and to practitioners seeking a principled framework for classroom design.

The remainder of the paper proceeds accordingly. Section 2 positions VCR in relation to adjacent traditions in mathematics education. Section 3 defines the model directly and clarifies its boundaries. Section 4 presents the programme theory and mechanism of action. Section 5 specifies the model as a design grammar through task ecology, sequencing rules, and error diagnostics. Section 6 describes the implementation architecture and lesson-plan validation. Section 7 presents three flagship illustrative exemplars. Section 8 discusses the conceptual, methodological, and practical significance of the model, and Section 9 concludes.

2. Positioning VCR in the Field

The Visualisation–Construction–Reasoning (VCR) Instructional Model is situated within a broad body of scholarship in mathematics education that has long emphasised the importance of representations, spatial thinking, geometric construction, conjecture, justification, and proof (Clements & Battista, 1992; Duval, 1995, 1998, 2006; Jones, 2012; Stylianides, 2007). The model does not emerge in isolation from these traditions, nor does it claim to replace them. Its contribution lies elsewhere. VCR is proposed as an instructional design grammar that translates several well-established insights of mathematics education into an explicit and replicable classroom architecture for school geometry.

2.1 Representational and semiotic foundations

A first point of contact for VCR lies in representational and semiotic perspectives that treat mathematical activity as inseparable from the use, interpretation, coordination, and transformation of representations (Duval, 1995, 1998, 2006). Geometry, perhaps more than many other school mathematical domains, depends on such coordination. Students must work across diagrams, verbal descriptions, symbolic notation, instrumented constructions, gestures, and sometimes physical models. A geometric idea is rarely encountered in one pure form; rather, it is mediated through representational systems that highlight certain features while suppressing others. From this standpoint, learning geometry cannot be reduced to memorising facts about figures. It necessarily involves learning how to see, translate, and stabilise meaning across representational forms.

VCR is fully aligned with this view. It treats geometry as representational work rather than as mere content coverage. Yet it moves a step further by asking an instructional question: once the centrality of representation is acknowledged, how should lessons be designed so that representation becomes a disciplined basis for mathematical action rather than a source of unexamined intuition? In many classrooms, diagrams are shown, copied, and discussed, but not sufficiently varied, tested, or interrogated. Students may encounter many figures without being helped to distinguish invariant properties from incidental appearance (Duval, 2006; Laborde, 2005). VCR responds to this by defining visualisation not as exposure to pictures, but as structured noticing across variation, comparison, transformation, and representational translation.

2.2 Visualisation as productive but fallible

A second point of contact lies with scholarship on visualisation and visual reasoning in mathematics. This body of work has shown that visual activity can support conjecture, insight, communication, pattern recognition, and generalisation, but also that visual experience alone does not guarantee mathematical validity (Arcavi, 2003; Duval, 2006; Fischbein, 1993). A figure may invite strong perceptual inferences that collapse when definitions, conditions, or counter-cases are introduced. School geometry therefore faces a persistent tension: visualisation is indispensable, but it is also insufficient when left under perceptual control.

VCR takes that tension as one of its starting points. The model treats visualisation as productive only when constrained by variation, comparison, and evidence. Learners must not only observe, but compare multiple cases, articulate what changes and what stays fixed, and connect visual patterns to explicitly stated properties. In other words, VCR does not celebrate visualisation in a general sense; it re-specifies visualisation as a disciplined instructional process whose goal is to reduce dependence on diagram illusion and strengthen property awareness.

2.3 Construction as epistemic work

A third theoretical neighbourhood for VCR is the tradition of geometric construction. Construction has historically occupied a central place in geometry because it embodies the relationship between definition, condition, operation, and object. To construct a geometric figure is not merely to draw it attractively; it is to produce an object that satisfies specified constraints through accepted means. Construction activities therefore have epistemic significance. They require learners to attend to what is given, what is possible, what is necessary, and how a result can be validated (Laborde, 2005).

VCR adopts this epistemic view of construction and makes it central. In the model, construction functions as the bridge between visual intuition and reasoned justification. It is the phase in which learners move from “I think this might be the case” to “I can generate or transform an object under conditions that test this claim.” Construction in VCR is never decorative and never terminal. It is paired with validation. Students are expected to check whether the object they have produced actually satisfies the defining constraints, and this checking becomes an important source of mathematical meaning.

2.4 Proof, justification, and classroom norms

A fourth and especially important point of contact is the literature on proof, proving, and mathematical justification in school contexts. This literature has demonstrated that proof should not be viewed as an elite or purely formal activity reserved for advanced learners. Rather, proof is a central mathematical practice through which statements are warranted, relationships are connected, and shared criteria of acceptability are established (de Villiers, 1990; Stylianides, 2007). It has also become clear that reasoning quality is shaped by classroom norms: what counts as an acceptable explanation, how teachers respond to informal arguments, and what kinds of evidence are legitimised (Harel & Sowder, 2007; Mariotti, 2006; Stylianides, 2007).

VCR is deeply consistent with this orientation. In the model, reasoning is not a ceremonial final stage in which students write formal proofs after “doing the real work” elsewhere. It is the phase in which geometric claims are made accountable through explicit reference to properties, conditions, and connected statements. The model acknowledges that students often experience a sharp discontinuity between informal explanation and formal proof. One of its instructional aims is to reduce that discontinuity by building intermediate supports: structured prompts, sentence frames, proof skeletons, justification chains, and teacher discourse moves that normalise evidence-based explanation.

2.5 What VCR adds

The distinct contribution of VCR becomes clearer when these traditions are viewed together. Representational perspectives explain why coordination across representations matters. Visualisation research clarifies the power and risk of perceptual activity. Construction traditions show why objects generated under constraints carry epistemic value. Scholarship on proof and classroom norms explains why reasoning must be treated as a disciplined and socially supported practice. Yet in ordinary classroom planning, these insights often remain distributed across separate literatures. Teachers and researchers may agree that all these things matter while still lacking a sufficiently precise way to organise them into an enactable geometry lesson system.

VCR is proposed as that operational link. It adds specification at five levels: it treats the relation among visualisation, construction, and reasoning as a designed instructional pathway; it specifies a task ecology; it defines sequencing rules; it identifies predictable error patterns with diagnostic responses; and it extends beyond theory into implementation architecture. In this sense, VCR stands in continuity with established research traditions while contributing a geometry-specific design grammar for classroom use.

Table 1

Positioning VCR Relative to Adjacent Frameworks

Adjacent framework / tradition	Primary concern	What it contributes to geometry education	What it does not specify fully for classroom design	What VCR adds
Representational / semiotic perspectives	Coordination across diagrams, symbols, language, and transformations	Shows that mathematical understanding depends on representational activity and translation	A lesson-level grammar for organising representational work into phase-linked classroom tasks	Converts representational insight into structured visualisation tasks, comparison routines, and translation moves
Visualisation research	Seeing structure, pattern, invariance, and spatial relations	Establishes the generative role of visual activity in conjecture and understanding	How visual work should be sequenced and constrained so that perception yields mathematically warranted insight	Reframes visualisation as disciplined noticing under variation, comparison, and evidence prompts
Construction-oriented approaches	Producing geometric objects under definitions and constraints	Highlights the epistemic role of construction in testing and instantiating geometric conditions	How construction should function as a bridge between conjecture and justification in a broader instructional sequence	Places construction centrally as validated constraint-based work linking seeing to justifying
Proof / proving / justification traditions	Establishing acceptable arguments and mathematical warrant	Clarifies the role of proof, acceptable explanation, and classroom norms of justification	How learners are instructionally prepared for proof through earlier visual and constructive phases	Builds reasoning from prior V and C work through justification chains, proof skeletons, and discourse scaffolds
General instructional cycles / activity models	Broad lesson flow, participation, engagement, and reflection	Provides useful generic pedagogical sequencing	Geometry-specific epistemic work, task ecology, phase-specific validation, and diagnostic design	Offers a domain-specific design grammar for geometry rather than a generic teaching cycle
VCR Instructional Model	Coordinated pathway from visualisation to construction to reasoning	Integrates representational, constructive, and justificatory work into one enactable model	Does not claim universal cognitive linearity; remains bounded to instructional design and implementation	Specifies task ecology, sequencing rules, diagnostic errors, classroom moves, and implementation architecture

3. Defining the VCR Instructional Model

3.1 Core definition

The Visualisation–Construction–Reasoning (VCR) Instructional Model is a structured, iterative, geometry-specific instructional design framework in which learners are guided through coordinated acts of visualisation, construction, and reasoning so that geometric understanding develops through representational noticing, constraint-based object work, and evidence-based justification.

In the model, visualisation refers to disciplined interpretation of figures, diagrams, spatial arrangements, and representational variations in ways that help learners identify relevant structure, distinguish invariant from incidental features, and formulate mathematically tractable conjectures. Construction refers to the production, transformation, or decomposition of geometric objects under explicit constraints, together with validation that the conditions have been satisfied. Reasoning refers to the explanation and justification of geometric claims through linked statements, accepted properties, warranted inferences, and communicable argument forms.

3.2 Unit of analysis

The unit of analysis in VCR is the instructionally designed geometry episode: a lesson segment or task sequence in which students are intentionally engaged in phase-linked representational, constructive, and justificatory work. This means that VCR is best analysed at the level of task design, classroom enactment, learner artefacts, and discourse moves taken together.

3.3 What VCR is and is not

VCR is not a visual learning-style approach, not an art-based method of drawing shapes, and not a proof-only pedagogy that begins with formalism. It is also not a claim about a universal psychological sequence through which all learners internally proceed in the same way. Rather, it is an instructional theory at the level of design and enactment. It specifies what kinds of tasks should be used, how they should be linked, what counts as progress from one phase to the next, and how teachers should respond when learners remain under perceptual rather than mathematical control.

3.4 Minimal criteria for identifying a VCR-aligned lesson

A lesson or lesson segment may be described as VCR-aligned only when the following minimum conditions are satisfied:

- it includes a genuine visualisation demand in which learners must notice structure, compare cases, identify invariants, or translate among representations.
- it includes a constraint-based construction or transformation demand.
- that construction includes explicit validation.
- it culminates in a reasoning demand requiring explanation, justification, or generalisation.
- the phases are instructionally linked rather than merely co-present.

Table 2

Minimal Criteria for Identifying a VCR-Aligned Lesson

Criterion	Required feature	What counts	What does not count
1. Visualisation demand	Learners must notice, compare, interpret, or identify invariants in figures or representations	Case comparison, invariant spotting, non-example analysis, translation across representations	Merely showing a diagram or asking students to look at a figure
2. Constraint-based construction	Learners must produce or transform a geometric object under explicit conditions	Constructing a figure from given conditions, decomposing and recomposing a shape, building a net or model under constraints	Free sketching, copying a teacher's drawing, decorative drawing
3. Validation layer	Learners must check whether the construction satisfies the intended constraints	Right-angle marking, measurement checks, adjacency checks, property verification, decomposition checking	Assuming the figure is correct because it looks neat or resembles the textbook figure
4. Reasoning demand	Learners must justify, explain, connect, or generalise from the work done	Justification chains, explanation prompts, proof skeletons, generalisation questions	Stating an answer without showing why it should be accepted
5. Phase linkage	Visualisation, construction, and reasoning must be instructionally connected	A conjecture generated in V leads to testing in C and is warranted in R	Three disconnected activities placed in one lesson without epistemic continuity

4. Programme Theory and Mechanism of Action

A model becomes citable and transferable when it states not only “steps” but a credible programme theory. VCR's mechanism can be summarised as follows.

First, visualisation generates structured conjectures by making invariants salient and by forcing representational translation and comparison across cases. Students are not asked merely what a figure looks like; they are asked what remains true when orientation, size, or configuration changes.

Second, construction tests and stabilises conjectures by requiring constraint specification and validation, thereby reducing reliance on diagram appearance. When learners must build or transform an object under explicit conditions, they can no longer rely exclusively on visual plausibility.

Third, reasoning consolidates and generalises by demanding explicit linkage of claims to properties and evidence and by exposing hidden assumptions. In turn, reasoning reshapes what students see in future visualisation. Thus, reasoning is both culmination and feedback.

This mechanism is designed to counter a common classroom failure mode: students conflate appearance with property, and teachers respond by delivering facts or proof templates. VCR intervenes earlier by reconstructing the ecology of tasks so that appearance-based claims are systematically challenged through variation, constraints, and justification norms.

Core Architecture of the VCR Model

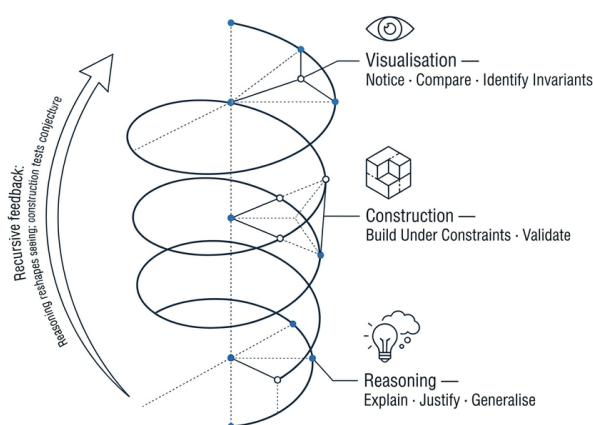


Figure 1. Core architecture of the VCR Instructional Model.

4.1 Logic model

The programme theory may be represented through a simple logic model. Inputs include lesson plans, teacher prompts, construction tools, representation sets, and assessment tasks. Activities include visualisation tasks, construction-with-validation tasks, and reasoning tasks. Immediate outputs include annotated diagrams, checked constructions, comparison records, and justification chains. Short-term outcomes include improved geometric discernment, reduced diagram illusion, stronger constraint awareness, and improved explanation quality. Longer-term intended outcomes include stronger geometry achievement, proof competence, and transfer to new geometric situations.

Programme Theory / Logic Model of VCR

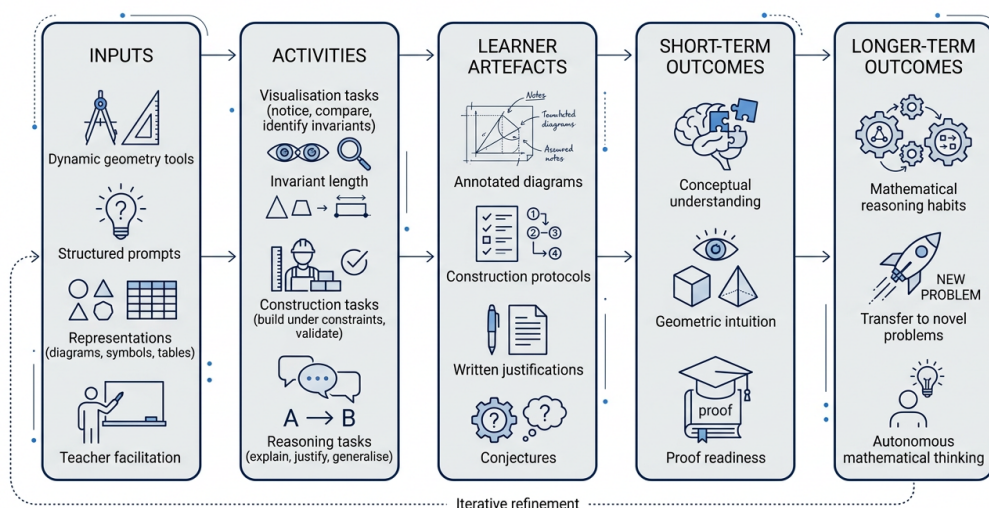


Figure 2. Programme theory / logic model of the VCR Instructional Model.

5. VCR as a Design Grammar for Geometry Instruction

VCR is not complete at the level of definition and mechanism alone. Its distinctiveness depends on whether it can specify how geometry lessons should actually be designed. This section presents VCR as a design grammar for geometry instruction.

5.1 Design principles

The grammar rests on six core principles.

Principle 1: Representation sets must be linked, not merely displayed. A representation set is linked when students must translate, coordinate, or reconcile verbal, diagrammatic, and symbolic forms.

Principle 2: Variation is mandatory for visualisation to become mathematical. Without variation, students can succeed by perceptual guessing. With variation, they must articulate what remains invariant.

Principle 3: Construction must include explicit constraint validation. Construction tasks require learners to show how each defining constraint has been satisfied through marks, measurements, or property-based checks.

Principle 4: Reasoning is scaffolded as chaining, not as final answers. Students are guided to build because–therefore chains, often with sentence frames or proof skeletons that reduce language load while preserving logical explicitness.

Principle 5: Error patterns are anticipated and designed against. Diagram illusion, constraint drift, circular reasoning, and premature generalisation are expected and planned for in advance.

Principle 6: Teacher role shifts across phases. During visualisation, the teacher frames attention; during construction, the teacher manages constraints and validation; during reasoning, the teacher stabilises norms of warrant and explanation.

5.2 Task ecology

A design grammar must specify not only principles but also the kinds of tasks through which those principles are realised. VCR is therefore built around a task ecology: a family of recurring task types that reliably elicit visualisation, construction, and reasoning. This taxonomy is intentionally compact. It is meant to be used by teachers, researchers, and reviewers. A VCR lesson is not required to include all families, but a VCR unit should include a planned distribution of them so that the model's mechanism can operate.

Table 3

Task Ecology of the VCR Instructional Model

Phase	Task family	Primary instructional purpose	Typical learner output
Visualisation	Property spotting	Direct attention to defining structure	Labelled or marked figure
Visualisation	Representation translation	Connect figure, language, and symbolic form	Figure-to-statement or statement-to-figure mapping
Visualisation	Case variation	Distinguish invariants from changing appearance	Comparison notes, invariant list
Construction	Constraint-based construction	Produce objects under stated conditions	Constructed figure with steps
Construction	Construction validation	Check whether conditions are satisfied	Validation marks, measurement checks
Construction	Decomposition/recomposition	Reveal hidden structure through partitioning or rearrangement	Partitioned or recomposed diagram
Reasoning	Justification chain	Link claim to property-based explanation	Ordered statements with reasons
Reasoning	Proof skeleton completion	Support structured argument formation	Completed proof framework
Reasoning	Generalisation	Extend beyond the immediate case	General statement with justification

5.3 Sequencing rules

The sequencing rules of VCR are intended to move learners from perceptual engagement to constraint-based construction and then to justificatory reasoning.

Rule 1: Begin with invariants before formal definition.

Rule 2: Use variation early to prevent diagram fixation.

Rule 3: Move from translation to construction when verbal–diagram alignment stabilises.

Rule 4: Embed validation immediately after construction, not at the end.

Rule 5: Shift from “what is true?” to “why must it be true?” using intermediate warrants.

Rule 6: Generalisation follows counterexample pressure, not repeated example confirmation alone.

Rule 7: Recycle visualisation after reasoning to deepen discernment.

5.4 Predictable error patterns and diagnostic responses

If VCR is to function as a replicable model rather than a charismatic teacher’s craft, it must specify predictable learner errors and corresponding responses. These responses are not generic corrections. They are epistemic moves that redirect authority from appearance to constraints and warrants. The predictable error logic and diagnostic response structure are explicitly embedded in the latest working draft of the paper and remain central to model replicability.

Table 4

Predictable Error Patterns and Diagnostic Responses in VCR

Phase	Error pattern	What it looks like	Diagnostic response
Visualisation	Diagram illusion	Student infers property from appearance alone	Ask for evidence, vary the case, redraw rotated or scaled
Visualisation	Trigger misidentification	Student focuses on irrelevant feature	Use “what changes/what stays” prompts
Construction	Constraint drift	Construction violates given conditions	Use explicit constraint checklist and peer validation
Construction	Over-trusting neatness	Claims correctness because the figure looks right	Require marks, checks, and property-based validation
Reasoning	Single-step explanation	“Because it is so” reasoning	Provide sentence frames and proof skeletons
Reasoning	Circular reasoning	Claim is used as its own justification	Ask for independent property support
Reasoning	Local generalisation	True for this diagram but not necessarily always	Use counterexample search and always/sometimes/never prompts

Figure 2. Typical Lesson Architecture — 45-Minute VCR Session

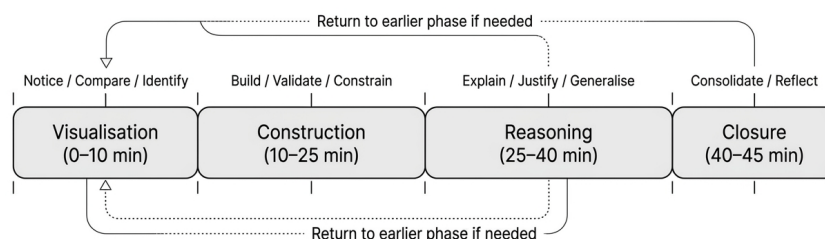


Figure 3. Typical lesson architecture for a 45-minute VCR session.

Figure 4. Activity-to-VCR Mapping Template

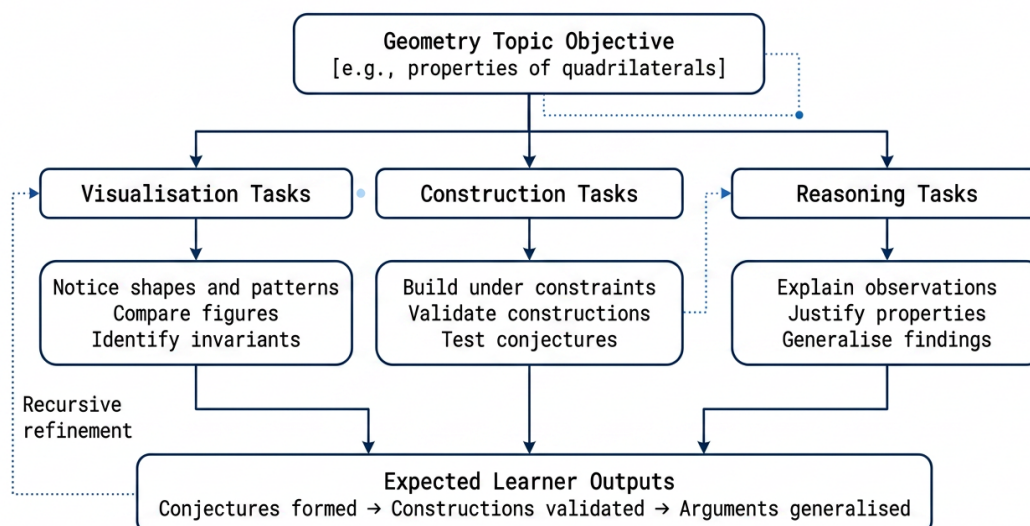


Figure 4. Activity-to-VCR mapping template for lesson planning.

6. Operationalisation Architecture of the VCR Model

A strong instructional model must show how it becomes enactable in ordinary classroom settings. In the present programme of work, VCR is operationalised through an 8-week instructional sequence, a set of 35 lesson plans, each of approximately 45 minutes, designed for bilingual classroom delivery and supported by multimodal teaching aids and representations. These architecture features are explicitly preserved in the latest uploaded paper form and are central to the model's packaging as a portable instructional system.

6.1 Dosage and delivery structure

The chosen dosage is pedagogically significant. VCR assumes that disciplined movement from visualisation to construction to reasoning cannot be stabilised through one-off exposure. Learners need repeated opportunities to encounter the V–C–R pathway across different topics, representations, and justificatory demands. A multi-week structure therefore allows the model to function as a pattern of instruction rather than an isolated event.

6.2 Lesson-plan syntax

A defining operational feature is that lesson activities are written in a syntax explicitly organised around the three processes. Lesson objectives are fulfilled through activities aligned with visualisation, construction, and reasoning. This mapping is crucial for fidelity measurement and teacher training because it makes the model observable.

6.3 Materials and multimodal resources

The implementation is multimodal by design. It includes charts, two-dimensional and three-dimensional models, pictorial figures, board work, and topic-specific representational supports. In VCR terms, these are not merely teaching aids; they are representational resources that enable translation, variation, constraint validation, and reasoning.

6.4 Teacher and student role architecture

The operationalisation assumes that the teacher is relatively more directive in visualisation and construction, while students become comparatively more active during reasoning. This corresponds to the model's theoretical commitment that reasoning must be normatively owned by learners. Teachers frame attention in V, guide constraints and validation in C, and press for warrant and explanation in R. The teacher–student role redistribution is clearly retained in the manuscript materials as a core active ingredient of the model.

6.5 Validation-oriented packaging

VCR is positioned as a standardised instructional package partly because the lesson-plan set was subjected to structured validation by both content experts and school teachers. The lesson plans were reviewed by a panel of 10 Expert Committee in Mathematics (ECM) members and 10 Elementary Mathematics School Teachers (EMST). Across the domains of format, content, language, and evaluation, the package received excellent composite-mean interpretations, indicating that the lesson plans were judged realistic, pedagogically coherent, linguistically appropriate, and assessment-aligned. This validation emphasis is one of the distinguishing features carried forward from the attached latest draft.

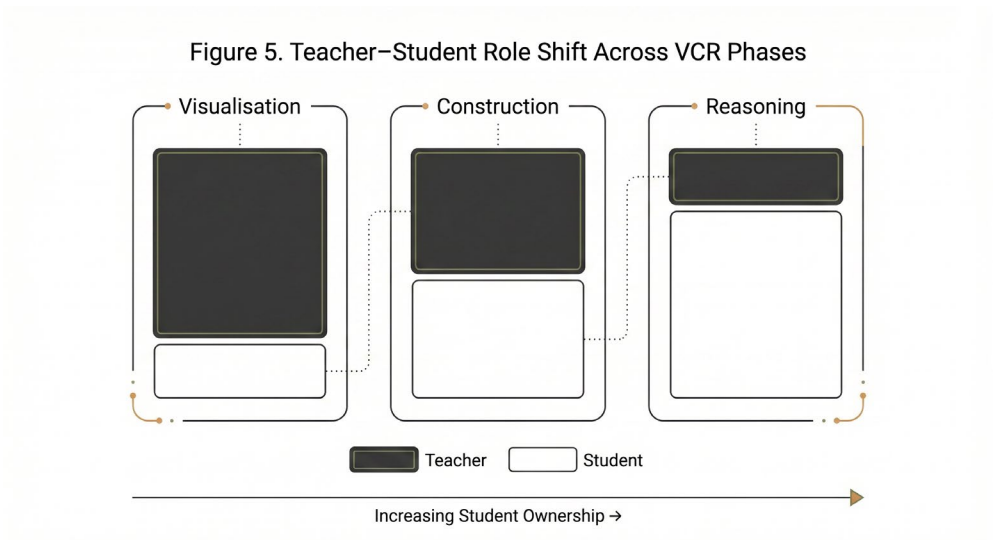


Figure 5. Teacher–student role shift across VCR phases.

Table 5
Implementation Architecture of the VCR Instructional Model

Component	VCR implementation specification
Duration	8 weeks
Total lesson plans	35
Approximate lesson length	45 minutes each
Delivery mode	Classroom-based, bilingual instructional delivery
Lesson structure	Phase-linked Visualisation → Construction → Reasoning sequence
Visualisation resources	Diagrams, varied cases, comparison prompts, worksheets
Construction resources	Ruler, compass, set square, models, foldables, guided tasks
Reasoning supports	Sentence frames, proof skeletons, justification chains
Teacher role architecture	Attention organiser in V; constraint guide in C; warrant-norm stabiliser in R
Student role architecture	Notice and conjecture in V; produce and validate in C; explain and justify in R
Validation basis	Expert and teacher panel review across format, content, language, evaluation

Table 6
Phase-wise “Look-fors” for VCR Enactment

Phase	Teacher look-fors	Student look-fors	Artefacts produced
Visualisation	Uses invariant prompts, contrast prompts, and representation linkage	Notices structure, compares cases, identifies invariants, translates between forms	Annotated diagrams, comparison notes, labelled figures
Construction	Restates constraints, checks step purpose, insists on validation	Constructs under conditions, checks correctness, uses marks and measures appropriately	Checked constructions, validation marks, construction records
Reasoning	Presses for reasons, property naming, and connected argument	Produces because–therefore chains, avoids circular reasoning, generalises cautiously	Explanation chains, proof skeletons, written justifications

7. Illustrative Exemplars of VCR in Geometry Instruction

A conceptual–method paper gains credibility when it shows how its design grammar operates in actual instructional episodes. To preserve focus, the present paper includes three main-text exemplars rather than a large activity bank. This retains the flagship-paper discipline while still demonstrating the topic range preserved in the attached working manuscript.

7.1 Exemplar 1: Altitude in a triangle

This exemplar addresses prototype dependence and diagram illusion. In the visualisation phase, students inspect acute, right, and obtuse triangles in varied orientations and decide which candidate segments count as altitudes. Prompts such as “Does an altitude always lie inside the triangle?” and “What remains invariant across these cases?” shift attention toward the defining condition of perpendicularity from a vertex to the opposite side or its extension.

In the construction phase, students draw altitudes in acute and obtuse triangles, including extension of the relevant side where necessary, and validate the construction by marking right angles. In the reasoning phase, they explain why an altitude in an obtuse triangle may lie outside the triangle without ceasing to be an altitude. The epistemic movement is clear: appearance-based concept recognition is replaced by property-based definition and justification.

7.2 Exemplar 2: Parallel lines cut by a transversal

This exemplar addresses the movement from visual pattern recognition to justified angle relations. In the visualisation phase, students compare multiple diagrams in which a transversal intersects two parallel lines at different orientations. They record which angle relations appear invariant across cases. In the construction phase, they construct parallel lines and several transversals, validate the condition of parallelism, and measure or mark angle pairs. In the reasoning phase, they complete justification chains or proof skeletons for corresponding or alternate angle relationships. The key pedagogical move is that observed regularity is converted into explicit warrant.

7.3 Exemplar 3: Area derivation through decomposition

This exemplar demonstrates that VCR is not restricted to identification or proof-type topics. In the visualisation phase, students inspect a trapezium and consider how it might be partitioned or rearranged into a more familiar area-bearing form. In the construction phase, they decompose and recombine shapes, mark preserved correspondences, and validate that area is preserved under the rearrangement. In the reasoning phase, they derive and justify a general rule for area. Here construction acts as a bridge from perceptual structure to generalisable mathematical argument.

Table 7

Summary of Main-Text Exemplars

Exemplar	Geometry topic	Main challenge addressed	Visualisation focus	Construction focus	Reasoning focus
Exemplar 1	Altitude in a triangle	Prototype dependence and diagram illusion	Invariant meaning across varied triangle forms	Constructing altitudes with perpendicular validation	Justifying why altitude may lie outside a triangle
Exemplar 2	Parallel lines and transversal	Pattern to proof-oriented relation	Detecting invariant angle relations	Constructing and validating conditions of parallelism	Explaining angle relations through accepted properties
Exemplar 3	Area derivation through decomposition	Formula as constructed relation	Seeing hidden structural equivalence	Decomposing and recomposing figures	Deriving and justifying a general area rule

8. Discussion

The VCR Instructional Model has been presented here as a geometry-specific instructional design grammar rather than as a generic teaching approach or universal developmental theory. Its significance can be interpreted at conceptual, methodological, and practical levels.

8.1 Conceptual contribution

Conceptually, VCR offers a sharpened way of thinking about geometry instruction by insisting that visualisation, construction, and reasoning must be treated as instructionally linked rather than pedagogically adjacent. The model

reframes geometry not primarily as a body of results, but as a domain of representationally mediated, constraint-sensitive, and justification-oriented activity. This is its strongest theoretical contribution.

Figure 6. Illustrative Exemplars of VCR

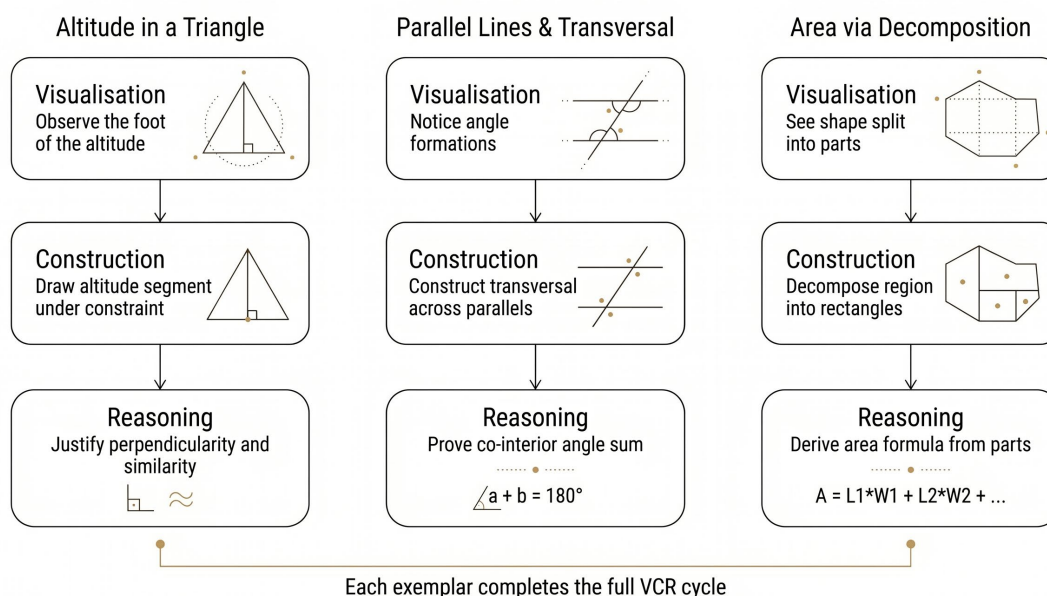


Figure 6. Illustrative exemplars of VCR in geometry instruction.

8.2 Methodological contribution

Methodologically, VCR provides a researchable framework with clear analytic boundaries. It defines a unit of analysis, specifies minimal lesson-alignment criteria, identifies a task ecology, articulates sequencing rules, and anticipates error patterns. It also centres learner artefacts such as annotated diagrams, checked constructions, and justification chains. These features make the model suitable for later implementation, fidelity, and outcome studies. In that sense, the present paper serves as a stable reference point for a broader research programme.

8.3 Practical contribution

Practically, VCR offers teachers and curriculum designers a principled structure for lesson planning. Instead of asking only what content is to be covered, the model asks what learners will notice, what they will construct or validate, and what they will be required to justify. It therefore provides a language for improving classroom design, teacher education, and materials development.

8.4 Boundary conditions and limitations

VCR is intentionally domain-specific. It is designed for school geometry and for topics in which figures, constraints, and justification are central. The present article is also a conceptual–method paper rather than an effect study. It establishes the model’s logic and architecture but does not report comparative outcome evidence. In addition, successful enactment depends on task quality, representational design, teacher mediation, and appropriate time allocation. Naming the three phases without preserving their epistemic functions would not constitute genuine implementation.

8.5 Research agenda

The model enables several lines of future research: effectiveness studies on geometry achievement and proof readiness; fidelity studies examining phase linkage and teacher role enactment; observation tools and coding manuals derived from the model’s look-fors; teacher-education interventions using VCR as a lesson-analysis framework; and curriculum adaptation studies across grade levels, institutional contexts, and language ecologies. Given the continuing vitality of proof and proving research in mathematics education (Stylianides et al., 2024), VCR is well positioned as a model that can connect classroom design to larger research agendas.

Table 8

Discussion Summary: Contribution, Limits, and Future Directions

Dimension	What VCR contributes	Key caution	Future direction
Conceptual	Coordinates seeing, building, and justifying in geometry	Not a generic pedagogy or universal cognitive theory	Extend dialogue with adjacent frameworks

Dimension	What VCR contributes	Key caution	Future direction
Methodological	Provides observable criteria, artefacts, and fidelity-ready structure	Present paper is conceptual–method, not effect-efficacy evidence	Conduct implementation and effect studies
Practical	Offers lesson-design grammar and teacher-role guidance	Depends on task quality and mediation	Develop toolkits and professional learning resources
Scope	Strong for school geometry	Not automatically generalisable to all mathematics	Explore principled adaptation across contexts

9. Conclusion

This paper has presented the Visualisation–Construction–Reasoning (VCR) Instructional Model as a geometry-specific instructional design grammar for school mathematics. The central argument has been that one of the enduring weaknesses of geometry teaching lies not simply in conceptual difficulty, but in the insufficient coordination of three essential forms of mathematical work: disciplined seeing, constraint-based construction, and warrant-based justification. VCR addresses that problem by specifying how these forms of work can be linked through a structured yet iterative instructional pathway.

The contribution of the paper lies in moving beyond a general endorsement of visual activity, hands-on engagement, or proof-oriented teaching in isolation. Instead, it has defined VCR as a coherent system with conceptual boundaries, a programme theory, design principles, a task ecology, sequencing rules, predictable error patterns, implementation architecture, and illustrative exemplars. It has also shown that the model exists as a reviewable instructional package supported by structured lesson-plan validation.

At the same time, the paper has remained bounded in its claims. It has not argued that VCR is a universal theory of cognition or a one-size-fits-all pedagogy for all branches of mathematics. Its claim is more precise and, for that reason, more defensible: in school geometry, learning can be made more coherent when instruction deliberately organises students' work around visualisation, construction, and reasoning as linked epistemic processes rather than disconnected classroom events.

In this sense, VCR is proposed as both a theoretical contribution and a practical architecture. It offers a language for analysing geometry lessons, a grammar for designing them, and a platform for extending research on classroom-based geometric understanding. As school mathematics continues to seek stronger ways of connecting intuitive engagement with mathematical warrant, the VCR model offers a disciplined pathway for moving geometry instruction from seeing, to building, to justifying.

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